Learn at Home Resource Packet – General Overview
Algebra I

This Next Generation Mathematics Learning Standards aligned packet of resources is designed for students and their parents who wish to support in-school learning with activities that can be done independently and/or with a partner at home. The packet includes five activities that support the major mathematical work of the grade with a particular focus on building grade level fluencies. In Algebra I, students' ability to fluently see structure in expressions, transform and manipulate polynomial expressions and equations, and use both symbolic and graphical representations of functions to understand and solve mathematical and real world problems are required as these support their ability to engage conceptually with important content of the year. These activities should each take 40-60 minutes (although many can be extended) and may be completed in any order.

How to use this guide - For each activity, you will find:

• information about the standards both content and practice that the activity supports;
• a description and/or instructions for the activity;
• materials required;
• one or more focus or discussion questions that will help deepen the learning of the activity;
• suggestions for extending or adjusting the activity.

Activity A
Kitchen Floor Tiles

From:
http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/215

Next Generation Mathematics Learning Standard(s)
Interpret the structure of expressions.
AI-A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★

Mathematical Practice
MP7: Look for and make use of structure.

Description

The purpose of this task is to give students practice in reading, analyzing, and constructing algebraic expressions, attending to the relationship between the form of an expression and the context from which it arises. The context here is intentionally thin; the point is not to provide a practical application to kitchen floors, but to give a framework that imbues the expressions with an external meaning.

Materials

- Pencil/Pen
- Colored Tiles

Focus question for discussion
What do you notice as related to the progression of the figures?

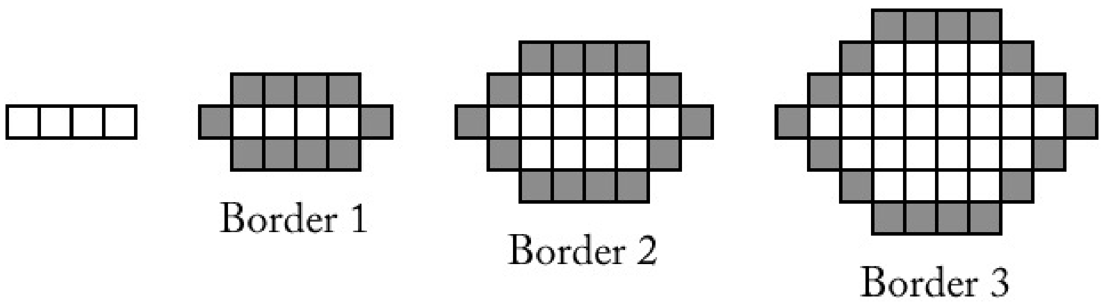
Extension
Participants who have completed the task should be convinced to continue. Encourage students to come up with different expressions that represent the figures.

Directions

- Analyzing and generalizing geometric patterns such as the one in this task may be familiar to students from work in previous grades, so part a may be a review of that process.
- Encourage participants to utilize different means through which they can record the progression of the figures (input/output, colored tiles, etc.)
- Provide participants with copies of the task and have them begin.

Task

Fred has some colored kitchen floor tiles and wants to choose a pattern using them to make a border around white tiles. He generates patterns by starting with a row of four white tiles. He surrounds these four tiles with a border of colored tiles (Border 1). The design continues as shown below:



Fred writes the expression $4(b-1)+10$ for the number of tiles in each border, where b is the border number, $b \geq 1$.

a. Explain why Fred's expression is correct.

b. Emma wants to start with five tiles in a row. She reasons, "Fred started with four tiles and his expression was $4(b-1)+10$. So if I start with five tiles, the expression will be $5(b-1)+10$. Is Emma's statement correct? Explain your reasoning.

c. If Emma starts with a row of n tiles, what should the expression be?

Activity B
Cash Box

From:
http://tasks.illustrativemathematics.org/content-standards/HSA/CED/A/tasks/462

Next Generation Mathematics Learning Standard(s)

Interpret functions that arise in applications in terms of the context.*

AI-A.CED.1 Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra II)

Notes:
- This is strictly the development of the model (equation/inequality).
- Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$
- Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where *a* is the first term and *r* is the common ratio.
- Inequalities are limited to linear inequalities.
- Algebra I tasks do not involve compound inequalities

AI-A.CED.2 Create equations and linear inequalities in two variables to represent a real-world context.

Notes:
- This is strictly the development of the model (equation/inequality).
- Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ *where* $a > 0$ *and* $b > 0$ $(b \neq 1)$.

AI-A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.

AI-A.CED.4 Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
e.g., Rearrange Ohm's law $V = IR$ to highlight resistance *R.*

Mathematical Practice(s)

MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.

Description

This tasks gives a teacher the opportunity to ask students not only for a specific answer of whether the dollar came from in the cash box or not, but for students to construct an argument as to how they came to their solution. While many tasks leave open an opportunity for a student to explain their answer, this task necessitates that explanation in order to settle the curiosity behind the context.

Materials

- Pen/Pencil
- Paper
- Calculator

Focus questions for discussion

- Students can begin with what they notice and what they wonder. Ask them.
- Encourage them to surface any immediate ideas that can be used to eliminate possible answers (e.g. “Were all 47 tickets sold to individuals?” Answer – No, 47 x 5 = 235)

Extension

This would be an opportune time to have students craft (and solve) their own tasks using the same scenario but with different dollar amounts. That would be one way to gauge their understanding of the task.

Directions

- Begin with a discussion that makes it clear to the students that as the facilitator, you are not looking for one specific answer. This task requires students to reason and provide explanations, whether verbally or in writing.
- Give participants and opportunity to formulate a plan for how they will pursue this task. Do not accept any answers. This phase of the segment is just for planning.
- Have the students begin the task below.

Task

Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said,

> ***I wonder whether the dollar belongs inside the cash box or not.***

The price of tickets for the dance was 1 ticket for \$5 (for individuals) or 2 tickets for \$8 (for couples). She looked inside the cash box and found \$200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

Activity C
Same Solutions?

From:
http://tasks.illustrativemathematics.org/content-standards/HSA/REI/A/tasks/613

Next Generation Mathematics Learning Standard(s)

Understand solving equations as a process of reasoning and explain the reasoning.

AI-A.REI.1a Explain each step when solving a linear or quadratic equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

AI-A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Note:
Algebra I tasks do not involve solving compound inequalities.

Mathematical Practice(s)

MP3: Construct viable arguments and critique the reasoning of others.
MP7: Look for and make use of structure.

Description

It is easy for students to lose sight of what it means for two equations to be equivalent they are trained to immediately start following a step-by-step procedure to solve an equation. The purpose of this task is to provide an opportunity for students to look for structure when comparing equations and to reason about their equivalence.

Materials

- Paper
- Pens/Pencils

Focus questions for discussion

- What does it mean for two equations to be equivalent?
- How might one go about determining whether or not two equations are equivalent without solving them?

Extension

Students can practice honing their skills with this standard by creating their own equations and determining whether or not they are equivalent. Start off by presenting students with expressions. After they have had practice with them, you can have them begin to use their abilities with equations.

Directions

- Before delving in, have participants share their preliminary observations. During this time period, they should explain fully how they reach the conclusions they reach (e.g. "I can see that equation 'a' is equivalent to equation 'e'. The only differences are (1) the coefficients in equation 'e' have been multiplied by two and (2) in equation 'e', the expressions on either side of the equal sign have been placed on the other side.
- Provide students with the task and have them begin working.

Task

Which of the following equations have the same solution? Give reasons for your answer that do not depend on solving the equations.

a. $x + 3 = 5x - 4$

b. $x - 3 = 5x + 4$

c. $2x + 8 = 5x - 3$

d. $10x + 6 = 2x - 8$

e. $10x - 8 = 2x + 6$

f. $0.3 + \frac{x}{10} = \frac{1}{2}x - 0.4$

Activity D
Operations with Rational and Irrational Numbers

From:
http://tasks.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/690

Next Generation Mathematics Learning Standard(s)

Use properties of rational and irrational numbers.

AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers.

a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots.

Note: Tasks include rationalizing numerical denominators of the form $a/\sqrt{b}$ where a is an integer and b is a natural number.

b.) Categorize the sum or product of rational or irrational numbers.

- The sum and product of two rational numbers is rational.
- The sum of a rational number and an irrational number is irrational.
- The product of a nonzero rational number and an irrational number is irrational.
- The sum and product of two irrational numbers could be either rational or irrational.

Mathematical Practice(s)

MP2: Reason abstractly and quantitatively.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Description

This task has students experiment with the operations of addition and multiplication, as they relate to the notions of rationality and irrationality. As such, this task perhaps makes most sense after students learn the key terms (rational and irrational numbers), as well as examples of each (e.g., the irrationality of $\sqrt{2}$, **π** etc.), but before formally proving any of the statements to be discovered in this task. Discussion of such proofs is taken up in other tasks.

Materials

- Pencil/Pen
- Scrap paper

Focus questions for discussion

• How does “always”, “sometimes”, and “never” work?
• Give students an example and ask, “Which category does this answer fit in?”

e.g., $|-7| + (-7)$ “always”, “sometimes” or “never” = zero?

Directions

- Have participants generate everyday examples within the categories “always”, “sometimes”, and “never.”
- Participants can work in small groups to generate ideas among themselves. However, participants can also generate ideas be themselves.
- Have participants explain rational and irrational numbers.
- Provide participants with the task.

Task

Experiment with sums and products of two numbers from the following list to answer the questions that follow:

$$5, \frac{1}{2}, 0, \sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}}, \pi.$$

Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true?

a. The sum of a rational number and a rational number is rational.

b. The sum of a rational number and an irrational number is irrational.

c. The sum of an irrational number and an irrational number is irrational.

d. The product of a rational number and a rational number is rational.

e. The product of a rational number and an irrational number is irrational.

f. The product of an irrational number and an irrational number is irrational.

Activity E
Paying the Rent

From:
http://tasks.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/581

Next Generation Mathematics Learning Standard(s)

AI-A.CED.1 Create equations and inequalities in one variable to represent a real-world context.
(Shared standard with Algebra II)

Notes:
- This is strictly the development of the model (equation/inequality).
- Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ where $a > 0$ and $b > 0$ $(b \neq 1)$.
- Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where *a* is the first term and *r* is the common ratio.
- Inequalities are limited to linear inequalities.
- Algebra I tasks do not involve compound inequalities.

Mathematical Practice(s)

MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.

Description

This simple conceptual task focuses on what it means for a number to be a solution to an equation, rather than on the process of solving equations.

Materials

- Pencil/Pen
- Scrap paper
- Graph Paper

Focus questions for discussion

- Based on the information provided, what are some preliminary conclusions you are able to draw? (Sample Response – "After x months, the funds will be depleted.")
- There is a constant amount being withdrawn, which reveals a linear relationship.

Directions

- To ensure that participants are clear on the task, have a volunteer paraphrase what the task is. Be sure the volunteer states that no other transactions take place in the account.
- Give participants two-minutes to draft a plan for approaching the task. Inform them that no solutions are required at this juncture.
- Provide participants with time to complete the task and have them begin.

Extension

Have students work on the same task. However, during the extension activity, there will also be $150 monthly deposits.

Task

A checking account is set up with an initial balance of $4800, and $400 is removed from the account each month for rent (no other transactions occur on the account).

a. Write an equation whose solution is the number of months, m, it takes for the account balance to reach $2000.

b. Make a plot of the balance after m months for for $m = 1, 3, 5, 7, 9, 11$ and indicate on the plot the solution to your equation in part (a).

Activity F
Allergy Medication

From: https://tasks.illustrativemathematics.org/content-standards/HSF/LE/A/2/tasks/2125

Next Generation Mathematics Learning Standards

Construct and compare linear, quadratic, and exponential models and solve problems.

AI-F.LE.2

Construct a linear or exponential function symbolically given:

i) a graph;

ii) ii) a description of the relationship;

iii) iii) two input-output pairs (include reading these from a table).

(Shared standard with Algebra II)

Note: Tasks are limited to constructing linear and exponential functions in simple context (not multi-step)

Mathematical Practice

M**P6** – Attend to precision.

Description

The purpose of the task is to help students become accustomed to evaluating exponential functions at non-integer inputs and interpreting the values. They do this by constructing a model to interpret a real-world situation with an emphasis on evaluating at non-integer values.

Materials

- Pencil/Pen
- Paper
- Calculator

Focus questions for discussion

- Do you predict that chemicals may leave a person's bloodstream at a consistent rate or at an inconsistent rate? Explain your reasoning.
- What are some factors that may determine how a body processes chemicals?

Directions

- Have students anticipate some of the outcomes and their reasoning before they delve deeply into the task.
- Give them time to go over the questions alone. Thereafter, those who would like to work with others should be allowed to do so.

Extension

Students may be interested in how the different systems of the body function over time. One efficient activity relates to pulse rate. Show them where to properly place their fingers to gauge their pulse rate. (They can place their index and middle fingers by their wrists, count the number of pulsations they pick up in ten seconds, and multiply that amount by 6, which would yield the number of beats per minute.) Next, have them do 20 jumping jacks and measure their pulse rates thereafter. This allows them to see the connection between exercise, heart rate, and pulse.

Task

Every day Brian takes 20 mg of a drug that helps with his allergies. His doctor tells him that each hour the amount of drug in his blood stream decreases by 15%.

a. Construct an exponential function of the form $f(t) = ab^t$, for constants a and b, that gives the quantity of the drug, in milligrams, that remains in his bloodstream t hours after he takes the medication.

b. How much of the drug remains one day after taking it?

c. Do you expect the percentage of the dose that leaves the blood stream in the first half hour to be more than or less than 15%? What percentage is it?

d. How much of the drug remains one minute after taking it?

Activity G
Pizza Place Promotion

From: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/B/tasks/578

Next Generation Mathematical Standards

AI-F.IF

Interpret functions that arise in applications in terms of the context.

Mathematical Practice

MP7 Look for and make use of structure.

Description

This task requires students to make interpretations regarding the prices of pizza on specifics dates. Students will use function notation and will construct expressions.

Materials

- Pencil/Pen
- Paper
- Calculator

Focus questions for discussion

- By examining the function, what is the lowest price of one whole pizza?
- Describe some of the information given that provided you with a starting point.

Directions

Students will need to analyze the data before determining the manner in which they plan to address the questions. Some students will find it beneficial to take note of the information from the problem that helped them get started.

Extension

Have students use other number values to address the same questions. They can plug in different numbers, solve the questions, and then share their questions with others.

Task

In order to gain popularity among students, a new pizza place near school plans to offer a special promotion. The cost of a large pizza (in dollars) at the pizza place as a function of time (measured in days since February 10th) may be described as

$$C(t) = \begin{cases} 9, & 0 \le t \le 3 \\ 9 + t, & 3 < t \le 8 \\ 20, & 8 < t < 28 \end{cases}$$

(Assume t only takes whole number values.)

a.

If you want to give their pizza a try, on what date(s) should you buy a large pizza in order to get the best price?

b.

How much will a large pizza cost on Feb. 18th?

c.

On what date, if any, will a large pizza cost 13 dollars?

d.

Write an expression that describes the sentence "The cost of a large pizza is at least A dollars B days into the promotion," using function notation and mathematical symbols only.

e.

Calculate $C(9) - C(8)$ and interpret its meaning in the context of the problem.

f.

On average, the cost of a large pizza goes up about 85 cents per day during the first two weeks of the promotion period. Which of the following equations best describes this statement?

- $\frac{C(13)+C(0)}{2} = 0.85$
- $\frac{C(13)-C(0)}{13} = 0.85$
- $\frac{C(13)}{13} = 0.85$
- $\frac{C(\text{Feb.}23)-C(\text{Feb.}10)}{13} = 0.85$

Activity H
Throwing Baseballs

From: http://tasks.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/1279

Next Generation Mathematical Standards

Interpret functions that arise in applications in terms of the context.

AI-F.IF.4 For a function that models a relationship between two quantities:

i) interpret key features of graphs and tables in terms of the quantities; and

ii) ii) sketch graphs showing key features given a verbal description of the relationship. (Shared standard with Algebra II)

Notes:

- Algebra I key features include the following: intercepts, zeros; intervals where the function is increasing, decreasing, positive, or negative; maxima, minima; and symmetries.
- Tasks have a real-world context and are limited to the following functions: linear, quadratic, square root, piece-wise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ *where* $a > 0$ *and* $b > 0$ $(b \neq 1)$

Analyze functions using different representations.

AI-F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
(Shared standard with Algebra II)

Note:
Algebra I tasks are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and exponential functions of the form $f(x) = a(b)^x$ *where* $a > 0$ *and* $b > 0$ $(b \neq 1)$.

Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others.
MP6 Attend to precision.

Description

It allows the students to compare characteristics of two quadratic functions that are each represented differently, one as the graph of a quadratic function and one written out algebraically. Specifically, we are asking the students to determine which function has the greatest maximum and the greatest non-negative root.

Materials

- Pencil/Pen
- Paper
- Calculator

Focus questions for discussion

- What are the different ways you could compare the maximums and roots of the two functions?
- What must you first do in order to compare the peak height that both baseballs reach?

Directions

- Since students are able to solve this problem algebraically or by graphing the functions, you should first have a conversation so that they reveal both methods to the teacher.
- As they discuss this, it will become apparent to some students that graphing both functions is the quicker of the two methods. After the discussion, have students begin working on the problems.

Extension

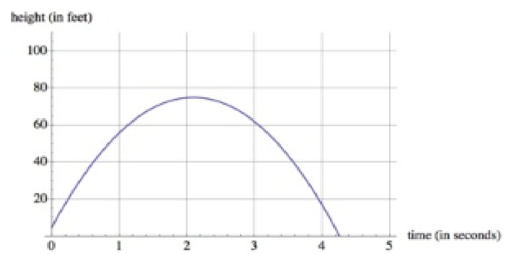
Have students create other scenarios, using two other baseball players who compare the heights of balls they have thrown. Students can solve their own problems and present them to others for solving.

Task

Suppose Brett and Andre each throw a baseball into the air. The height of Brett's baseball is given by

$$h(t) = -16t^2 + 79t + 6,$$

where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below:



Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.

a. Who is right?

b. How long is each baseball airborne?

c. Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

Activity I
Seeing Dots

From: http://tasks.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/21

Next Generations Mathematics Learning Standards

AI-A.SSE.1

Interpret expressions that represent a quantity in terms of its context. ★

AI-A.SSE.2

Recognize and use the structure of an expression to identify ways to rewrite it.
(Shared standard with Algebra II)

e.g., $x^3 – x^2 - x = x(x^2 - x - 1)$
$53^2 – 47^2 = (53 + 47) (53 - 47)$
$16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6) (4x - 6) = 4(2x + 3) (2x - 3)$ or
$16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3) (2x - 3)$
$-2x^2 + 8x + 10 = -2(x^2 – 4x – 5) = -2(x - 5) (x + 1)$
$x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1)$

Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form $ax^2 +bx + c$ with a lead coefficient of 1, or a combination of methods to factor completely. Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.

Mathematical Practices

MP2 Reason abstractly and quantitatively
MP7 Look for and make use of structure

Description

The purpose of this task is to identify the structure in the two algebraic expressions by interpreting them in terms of a geometric context. Students are asked to notice a pattern and connect the pattern to the algebraic representation.

Materials

- Pencil/Pen
- Paper

Focus question for discussion

Can you use factoring to assist with this task? If so, how?

Directions

- Students may begin by finding equivalence algebraically. Encourage those students to relate the components of their expressions to the illustrations. Have students relate the coefficients, variables, and exponents to the diagram.
- Encourage to think geometrically as well as algebraically.

Extension

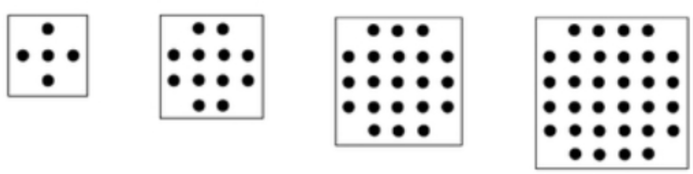
Students who have been successful on this task should be encouraged to come with their own illustrations and dot arrangements. They should be encouraged to create scenarios, solve them, and then challenge others to solve their made up examples.

Task

Consider the algebraic expressions below:

$$(n+2)^2 - 4 \qquad \text{and} \qquad n^2 + 4n.$$

a.
Use the figures below to illustrate why the expressions are equivalent:



b. Find some ways to algebraically verify the same result.

Task J
Basketball

From: http://tasks.illustrativemathematics.org/content-standards/HSA/REI/A/2/tasks/702

Next Generations Mathematics Learning Standards

AI-A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Note: Algebra I tasks do not involve solving compound inequalities.

AI-A.CED.1 Create equations and inequalities in one variable to represent a real-world context. (Shared standard with Algebra II)

Notes:

- This is strictly the development of the model (equation/inequality).
- Limit equations to linear, quadratic, and exponentials of the form $f(x) = a(b)^x$ *where* $a > 0$ *and* $b > 0$ $(b \neq 1)$.
- Work with geometric sequences may involve an exponential equation/formula of the form $a_n = ar^{n-1}$, where *a* is the first term and *r* is the common ratio.
- Inequalities are limited to linear inequalities.
- Algebra I tasks do not involve compound inequalities.

Mathematical Practices

MP

MP

Description

This task provides a simple but interesting and realistic context in which students are led to set up a rational equation (and a rational inequality) in one variable, and then solve that equation/inequality for an unknown variable.

Materials

- Pencil/Pen
- Scratch Paper

Focus questions for discussion

- Begin by having students explain how they would determine the percentage of games someone has won.
- To begin, of the information that is provided, what could we use to answer the first question? (Be sure that students understand that the "new" total will be more than the thirty games he has already played.

Directions

- Begin with a basic, brief conversation about the task.
- As students go through each question, some answers will become apparent to them. (e.g., Question "c" – Chase has already lost winning 100% of his games is not possible.)
- Some students may want to begin using trial and error, which may work in the beginning but as the tasks become more challenging, students will see that another course of action is needed.

Extension

Have students come up with scenarios for questions "e" and "f".

Task

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

a. How many games would Chase have to win in a row in order to have a 75% wining record?

b. How many games would Chase have to win in a row in order to have a 90% winning record?

c. Is Chase able to reach a 100% winning record? Explain why or why not.

d. Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?