

## **Learn at Home Resource Packet – General Overview**

### **HS Geometry**

This Next Generation Mathematics Learning Standards aligned packet of resources is designed for students and their parents who wish to support in-school learning with activities that can be done independently and/or with a partner at home. The packet includes five activities that support the major mathematical work of the grade with a particular focus on building grade level fluencies. In Geometry, students' ability to fluently use triangle congruence and similarity, visualize rigid and similarity transformations, and use geometric representations as a tool is required as it supports their ability to engage conceptually with important content of the year. These activities should each take 40-60 minutes (although many can be extended) and may be completed in any order.

**How to use this guide** - For each activity, you will find:

- information about the standards both content and practice that the activity supports;
- a description and/or instructions for the activity;
- materials required;
- one or more focus or discussion questions that will help deepen the learning of the activity;
- suggestions for extending or adjusting the activity.

## Activity A

# Analyzing Congruence Proofs

From <http://map.mathshell.org/materials/download.php?fileid=1302>

### *Next Generation Mathematics Learning Standard(s)*

**Understand congruence in terms of rigid motions.**

#### **GEO-G.CO.7**

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

#### **GEO-G.CO.8**

Explain how the criteria for triangle congruence (ASA, SAS, SSS, **AAS and HL (Hypotenuse Leg)**) follow from the definition of congruence in terms of rigid motions.

**Prove theorems involving similarity.**

**GEO-G.SRT.5** Use congruence and similarity criteria for triangles to:

**GEO-G.SRT.5a** Solve problems algebraically and geometrically and to prove relationships in geometric figures.

**GEO-G.SRT.5b** Prove relationships in geometric figures.

Notes: ASA, SAS, SSS, AAS, and Hypotenuse-Leg (HL) theorems are valid criteria for triangle congruence.  $AA\sim$ ,  $SAS\sim$ , and  $SSS\sim$  are valid criteria for triangle similarity.

**This standard is a fluency recommendation for Geometry. Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.**

### *Mathematical Practice(s)*

MP3: Construct viable arguments and critique the reasoning of others.

### **Description**

In this activity students read description of shared properties of two triangles and determine if the two triangles must be congruent, or if two triangles can exist that not congruent, yet still share the properties listed. Students use sketches, constructions, examples and counterexamples to generate and support argument.

### **Materials**

- Pencil/Pen
- Ruler
- Protractor
- Compass
- Triangle property cards (below)
- Scrap paper
- Optional: interactive geometry software (a free option is GeoGebra, [www.geogebra.org](http://www.geogebra.org))

### **Focus questions for discussion**

- Is there a way to draw these triangles so they're not congruent? How do you know?
- Does it make any difference where the sides go and where the angle goes?
- Based on your examples, what conclusions did you reach?
- How would you explain your conclusion in writing?
- Is there an example or two that would make your argument clearer?
- Can you explain your diagram to the reader?

### **Extension**

Try to create a list of situations and information that give unique triangles (where there is only one triangle that can be drawn with the specifications given).

- What combination of sides and/or angles give unique triangles?
- What is the minimum amount of information that can give a unique triangle?
- What is the maximum amount of information that would allow for more than one triangle?

### **Directions**

1. Read through some of the property cards.
2. Choose one that makes the most sense to you.

3. Re-read the card and draw some examples of pairs of triangles that have the properties stated on the card.
4. Decide whether the two triangles **MUST** be congruent, record an answer, and an explanation:
  - If you decide that the triangles do not have to be congruent draw an example and explain why?
  - If you decide that the triangles must be congruent, try to write a convincing proof that explains why.
5. Repeat by choosing a different card to evaluate.

## Cards: Must the Two Triangles be Congruent?

1.  One side of Triangle A is the same length as one side of Triangle B.	2.  Two sides of Triangle A are the same lengths as two sides of Triangle B.	3.  Three sides of Triangle A are the same lengths as three sides of Triangle B.
4.  One side of Triangle A is the same length as one side of Triangle B and one angle in Triangle A is the same size as one angle in Triangle B.	5.  Two sides of Triangle A are the same lengths as two sides of Triangle B and one angle in Triangle A is the same size as one angle in Triangle B.	6.  Three sides of Triangle A are the same lengths as three sides of Triangle B. and one angle in Triangle A is the same size as one angle in Triangle B.
7.  One side of Triangle A is the same length as one side of Triangle B and two angles in Triangle A are the same sizes as two angles in Triangle B.	8.  Two sides of Triangle A are the same lengths as two sides of Triangle B and two angles in Triangle A are the same sizes as two angles in Triangle B.	9.  Three sides of Triangle A are the same lengths as three sides of Triangle B. and two angles in Triangle A are the same sizes as two angles in Triangle B.

## Activity B

# Transforming 2-D Figures

From <http://map.mathshell.org/materials/lessons.php?taskid=524&subpage=concept>

### ***Next Generation Mathematics Learning Standard(s)***

**Experiment with the transformations in the plane.**

**GEO-G.CO.2** Represent transformations **as geometric functions** that take points in the plane as inputs and give points as outputs. Compare transformations that preserve distance and angle **measure** to those that do not.

Note: Instructional strategies may include drawing tools, graph paper, transparencies and software programs.

### ***Mathematical Practice(s)***

MP1: Make sense of problems and persevere in solving them.. MP7:

Look for and make use of structure.

### ***Description***

In this activity students will build there understanding of geometric transformations by using different representations including words, drawings, and algebraic functions.

### ***Materials***

- Pencil/Pen
- Scissors
- Glue stick
- Card set; Transformations
- Scrap paper
- Optional: interactive geometry software (a free option is GeoGebra, [www.geogebra.org](http://www.geogebra.org), or Desmos, [www.desmos.org](http://www.desmos.org))

### ***Focus questions for discussion***

- Can you explain how you know this is a rotation? (or reflection or translation) □ How can you get the coordinates of the new point form the old point?
- What information do you need to describe transformation?

### ***Extension***

Students can make up their own transformations starting from any of the four representations on the card. Students may also want to try dilations and vertical scaling.

## Directions

1. In this activity you are also going to be describing transformations, as well as being given some descriptions of transformations to work with. When we describe a transformation using words we need to make sure that we include all the information required to describe the transformation exactly. Some transformations can be described by just one piece of information whereas some transformations need more than one piece of information to describe them. Look at the chart and discuss with a partner or parent:

Describing Transformations	
<b>A: Rotation</b>	1. A line 2. A vertical distance 3. An angle
<b>B: Reflection</b>	4. An axis 5. A direction 6. A center
<b>C: Translation</b>	7. A horizontal distance 8. A scale factor

Projector Resources

Transforming 2D Figures

P-1

- Which pieces of information (listed with numbers) are needed for each of the transformations (listed with letters)?
- When would you use an axis to describe a transformation?
- When would we use a scale factor to describe a transformation?

2. When a figure is transformed some or all of the points on the figure move to a new position, and every point on the original figure can be mapped to a new point on the translated figure with an algebraic function. Discuss the following two scenarios with a partner or a parent:

- Why can Scenario 1 be described by  $(x,y) \rightarrow (x+3,y)$  ?
- Why can Scenario 2 be described by  $(x,y) \rightarrow (2x,2y)$  ?

### Algebraic Notation Scenario (1)

If the  $x$  co-ordinates for all the points are increased by three how could we describe this?

$$(x,y) \rightarrow (\underline{\quad}, \underline{\quad})?$$

Projector Resources

Transforming 2D Figures

P-2

### Algebraic Notation Scenario (2)

If both the  $x$  and  $y$  co-ordinates are doubled how could we describe this?

$$(x,y) \rightarrow (\underline{\quad}, \underline{\quad})?$$

Projector Resources

Transforming 2D Figures

P-3

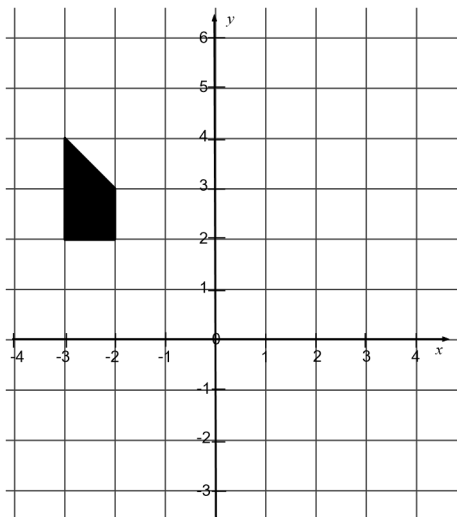
3. Cut out the Transformations card set. Each card has some missing information, either a algebraic description, description in words, the original figure, or the transformed image. Work with a partner or parent to complete the missing information and sort the card into piles by deciding if each is a rotation, reflection, or translation.
- Take turns to complete a card. Each time you do this; explain your thinking clearly and carefully.
  - Your partner should then either explain your reasoning again in his or her own words, or challenge the reasons you gave.
  - It is important that everyone in the group understands the transformation descriptions and figure positions – if not, ask questions.
  - Once you are all agreed on the completion of a card, decide weather it is a rotation, reflection, or translation.



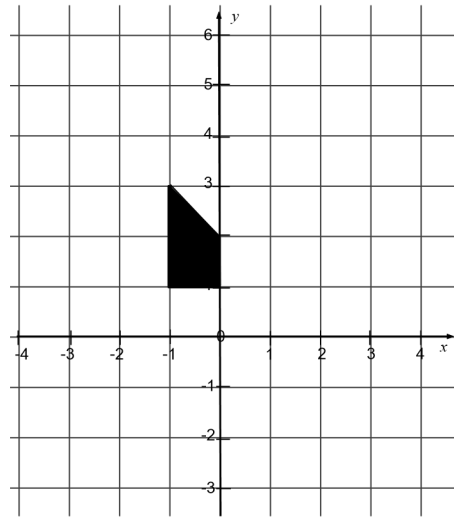
# Figure to Figure

1. The figure below is transformed to give a new image:

**Before:**



**After:**



Describe in words a single transformation that maps the original figure to the new image.

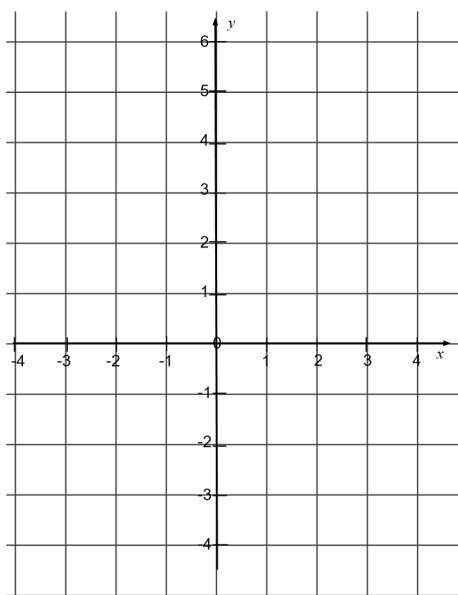
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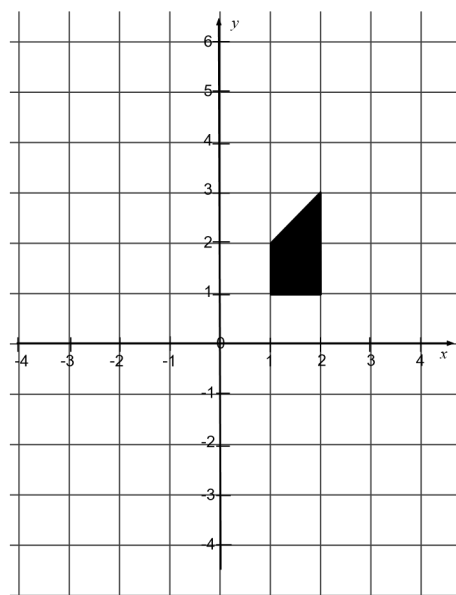
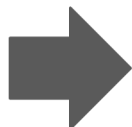
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2. A figure is reflected over the line  $y = -1$  to give the image below. Complete on the blank grid the position of the original figure before the transformation:

**Before:**

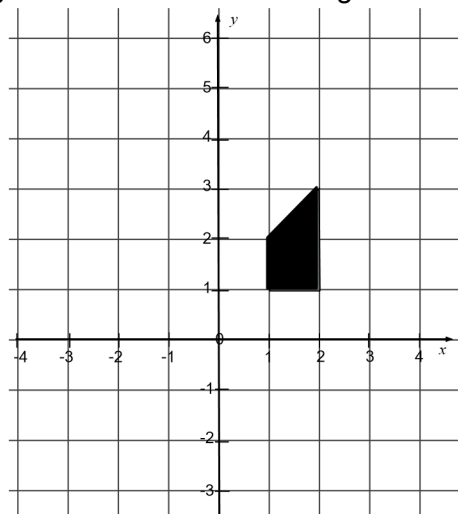


**After:**

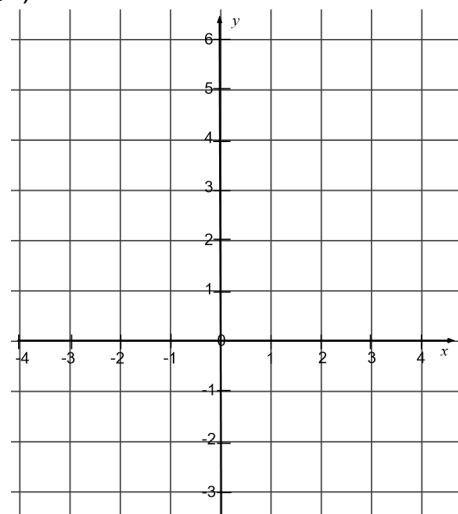


3. The figure below is rotated through  $90^\circ$  clockwise around  $(0,0)$ :

**Before:**



**After:**



Complete on the blank grid the image after the transformation and describe the effect of the transformation on  $(x,y)$ :

$$(x,y) \rightarrow (\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$$

Explain your answer:

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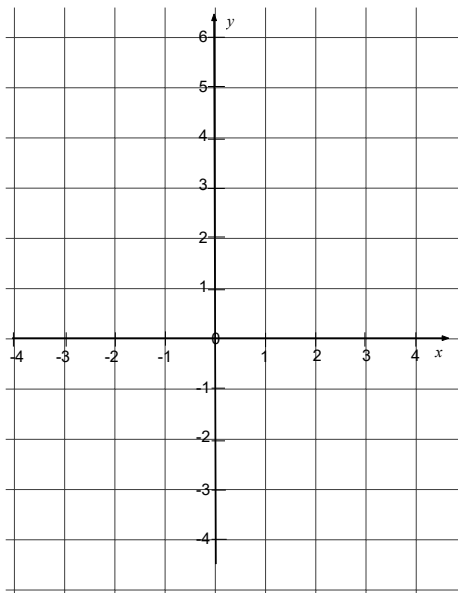
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## Card Set: Transformations (1)

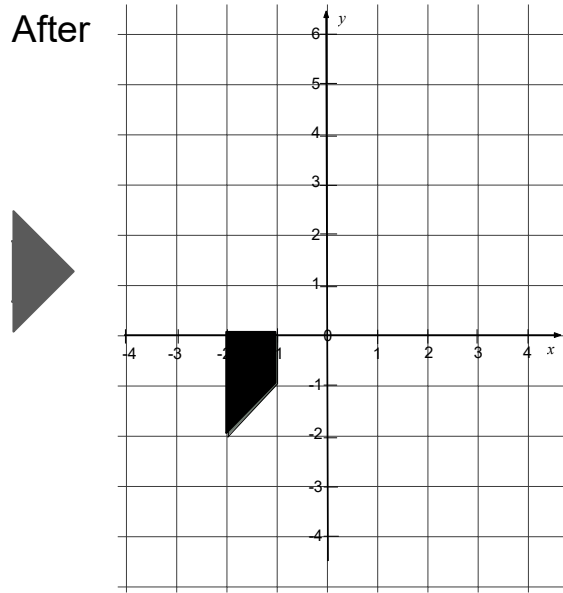
$$T_1(x,y) \textcircled{R}(\_,\_)$$

Reflection over the  $x$ -axis

Before



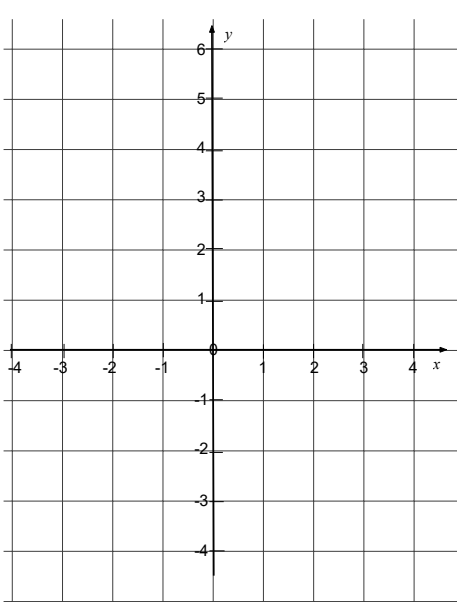
After



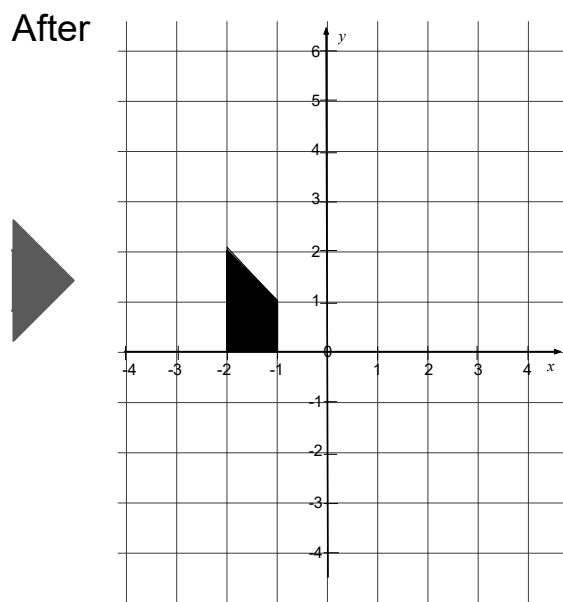
$$T_2(x,y) \textcircled{R}(\_,\_)$$

Translation -2 units vertically

Before



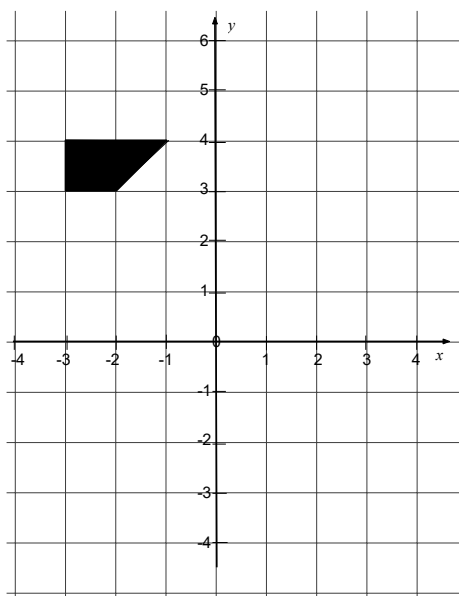
After



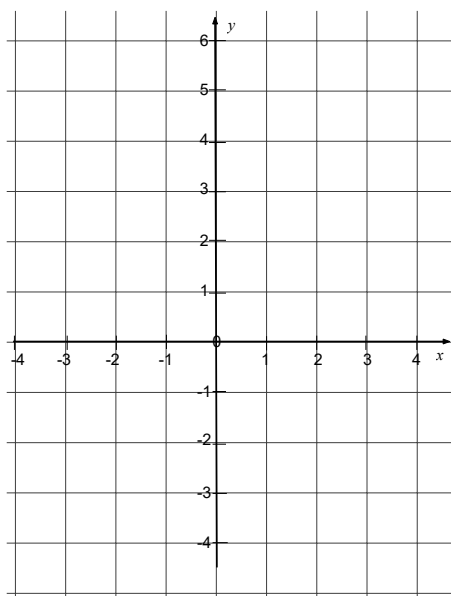
## Card Set: Transformations (2)

$$T_3(x,y) \textcircled{R} (-x,-y)$$

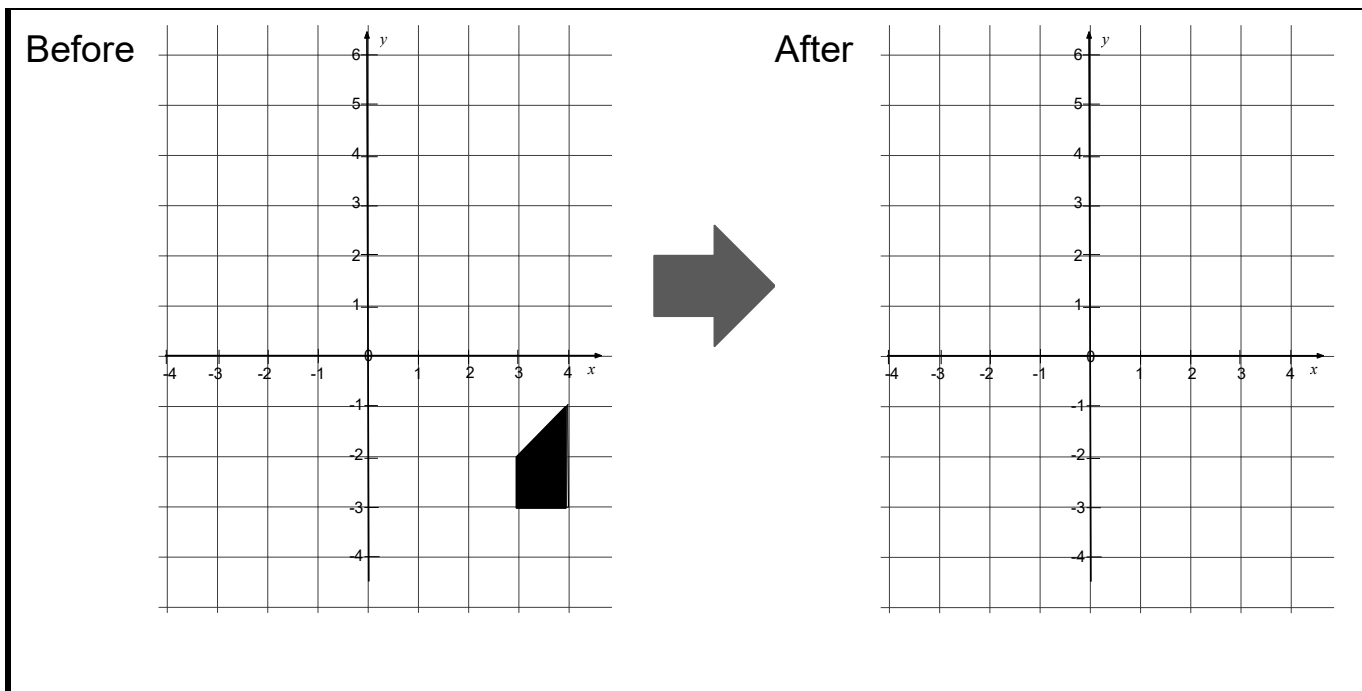
Before



After



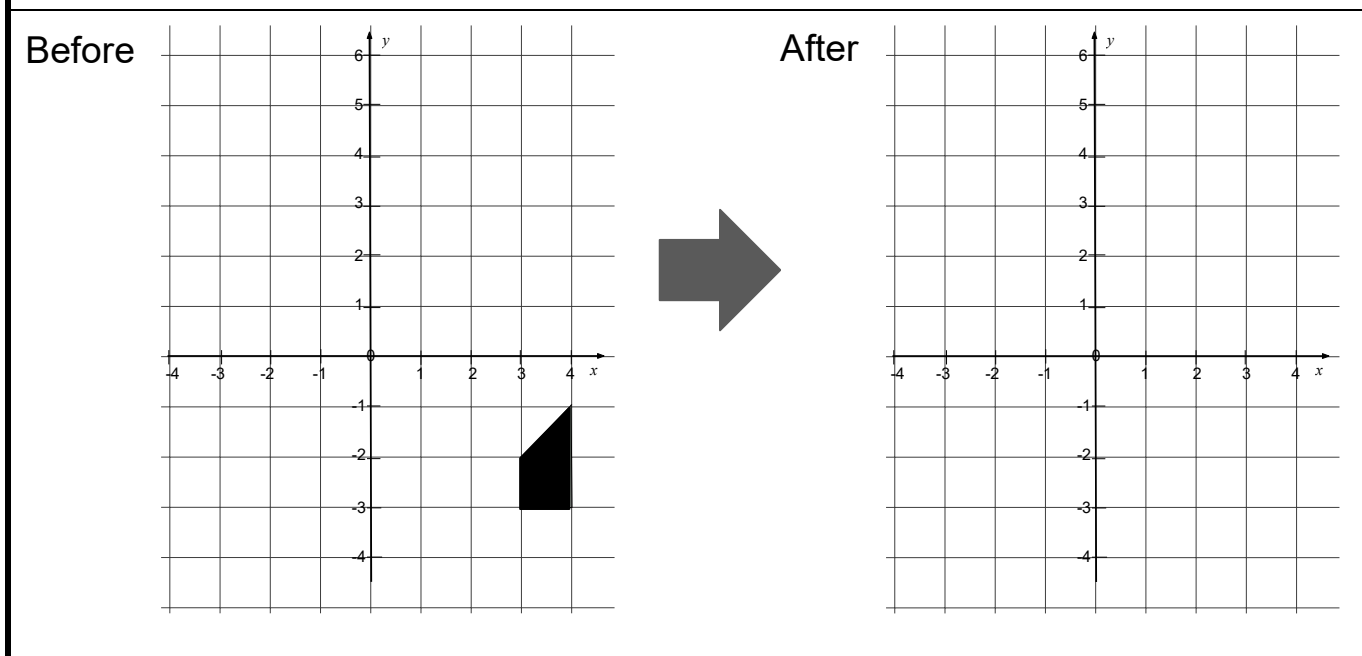
$$T_4(x,y) \textcircled{R} (x-2,y+4)$$



### Card Set: Transformations (3)

$$T_5(x, y) \textcircled{R} (\_, \_)$$

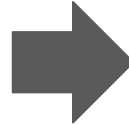
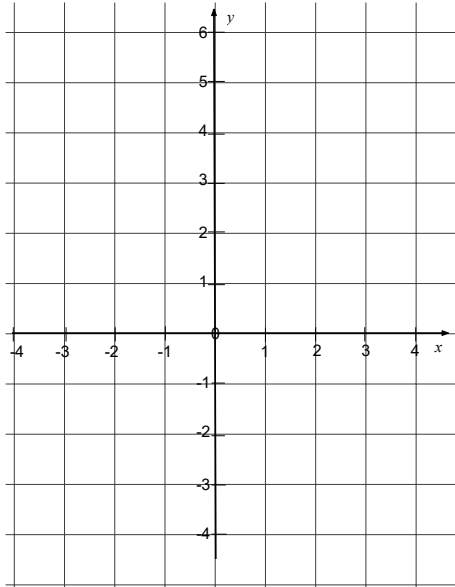
Reflection over the  
line  $y = x$



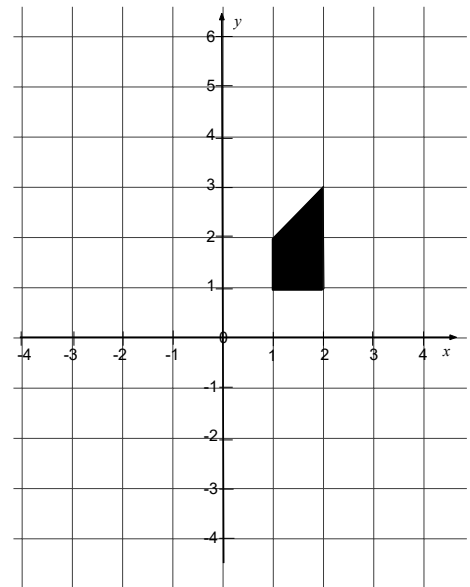
$T_6(x,y) \textcircled{R}(\_,\_)$

Rotation through  $90^\circ$   
clockwise around  $(1,1)$

Before



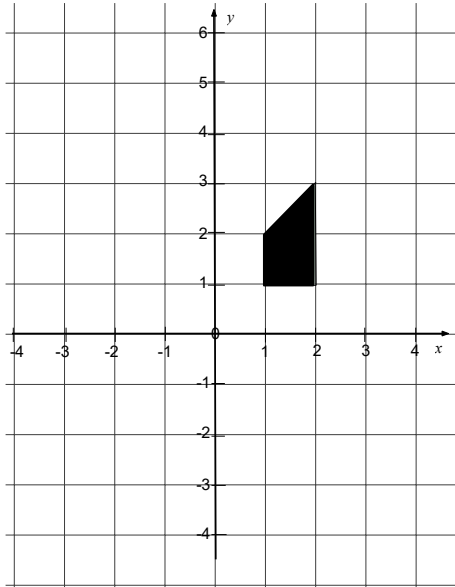
After



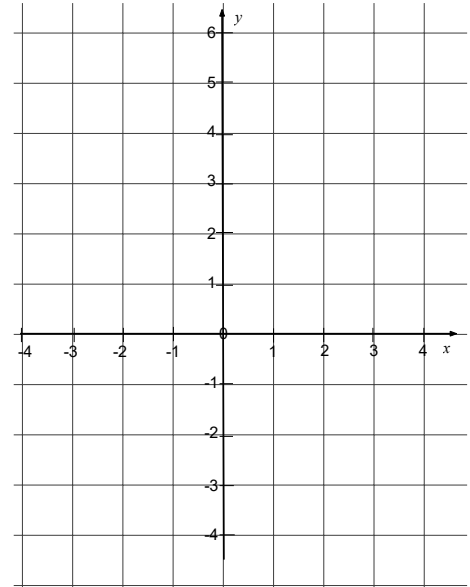
## Card Set: Transformations (4)

$$T_7 (x,y) \rightarrow (-x+2,y)$$

Before



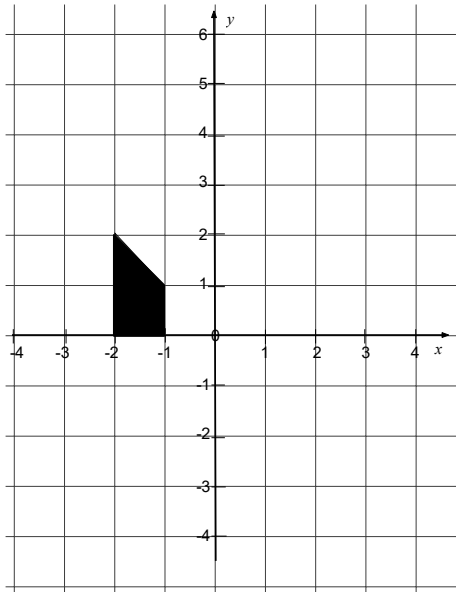
After



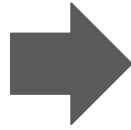
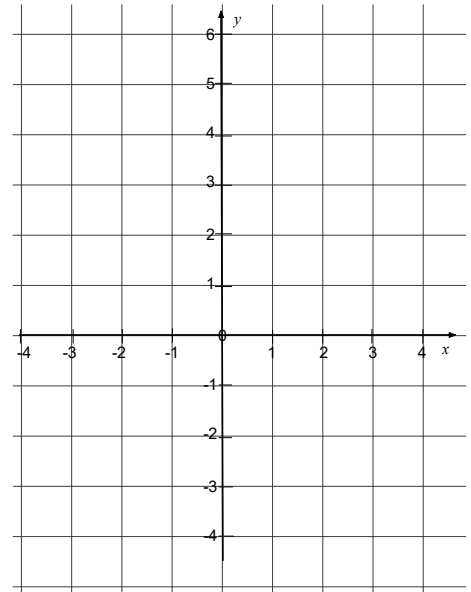
$$T_8 (x,y) \rightarrow (\_,\_)$$

Rotation through  $90^\circ$   
counterclockwise around  
(0,0)

Before



After





## Activity C

# Running Around a Track I

From: <https://tasks.illustrativemathematics.org/content-standards/tasks/1127>

### ***Next Generation Mathematics Learning Standard(s)***

**Understand similarity in terms of similarity transformations.**

**GEO-G.MG.1** Use geometric shapes, their measures, and their properties to describe objects. ★

**GEO-G.MG.2** Apply concepts of density based on area and volume of geometric figures in modeling situations. ★

**GEO-G.MG.3** Apply geometric methods to solve design problems. ★

**Note:** Applications may include designing an object or structure to satisfy constraints such as area, volume, mass and cost.

### ***Mathematical Practice(s)***

MP4: Model with mathematics.

### ***Description***

In this task, students use geometry to find the perimeter of a racetrack. The track is divided into lanes and as a result, the distance ones runs around the track is influenced by the lane a runner occupies and by the runner's position while in the lanes. Students are asked to ascertain the distance one runs if he/she is in the inner most lane of the track. Thereafter, they must ascertain the distance one runs if he/she is in the outermost portion of that inner lane. Although it is a 400-meter track, students may be surprised when their calculations do not yield 400 meters but rather a smaller number. Facilitators may wish to stop at this point for a discussion on how this can be. Hopefully, the facilitator would be able to lead the students to understand that a runner would be disqualified if his/her steps were out of bounds (on the border of the track).

### ***Materials***

- Pencil/Pen
- Table of mathematical formulas
- Scrap paper
- Calculators
- Ball of string

**Focus questions for discussion**

- How can we use geometric shapes, their measures, and their properties to describe objects?
- In which ways can we apply concepts of density based on area and volume of geometric figures in modeling situations?
- How can we apply geometric methods to address the design of a racetrack?

**Extension**

- Provide participants sheets of paper, containing drawings of circles of different sizes (e.g. lids from containers; outlines from the bottom of cups). Instruction students to fold one end of the circle over the other so that the segments overlap accurately. After unfolding the cut-outs, they now have circles that have a line of symmetry, representing the diameter. Have participants use pieces of string to measure the circumference of each cut-out they have. Students should then fill in a chart similar to this:

Circumference of the Circle (cm)	Length of the Diameter (cm)	Circumference ÷ Diameter

This activity helps students construct the meaning of pi. After the sections of the chart are filled in, participants will see that each time circumference is divided by diameter, the result will be close to 3.14.

- Students are often instructed to use the fraction  $\frac{22}{7}$  when approximating for pi. Have participants practice dividing other numerators by denominators and seeing how close they can come to 3.14.

**Directions**

**Introduction**

1. Have a discussion with the students regarding the Olympics (track & field; every four years). You may even chose to show them short videos of long-distance runners circling a racetrack. Students will most likely point out that the runners are vying for the inside position and thus, the shortest distance around the track.
2. Call students' attention to the innermost and outermost lanes. Lead them to understand what occurs when one is running on the straight portions of the track as opposed to what

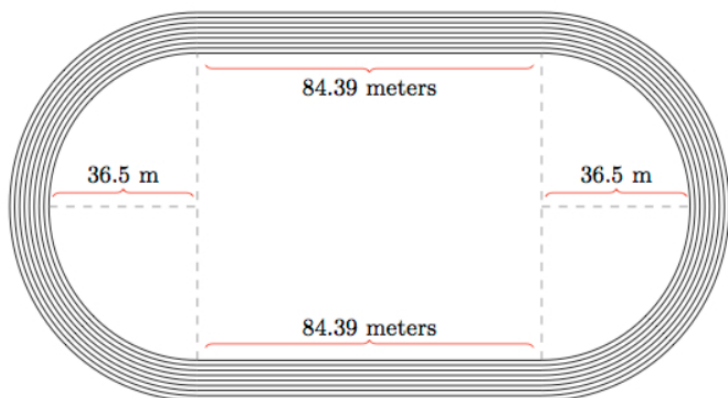
occurs when one is running on the curved portions of the track. (Curved portions - the further away from the inner lane one is, the greater the distance being run.)

3. Students can now begin addressing the following questions:

- a) What is the perimeter of the track, measured on the innermost part of the first lane?
- b) Each lane on the track is 1.22 meters wide. What is the perimeter of the track measured on the outermost part of the first lane?
- c) In order to run the intended 400-meters in a lap, how far away from the inside of the first lane would a runner need to be?

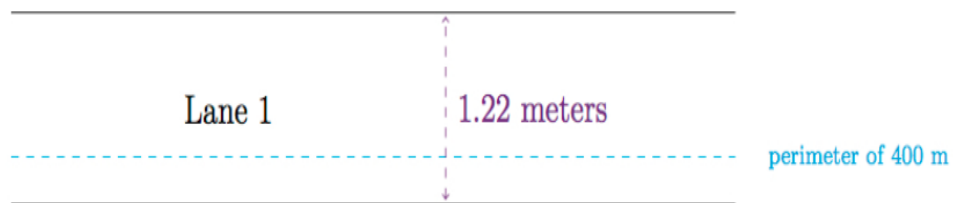
## Task

An Olympic 400 meter track is made up of two straight sides, each measuring 84.39 meters in length, and two semi-circular curves with a radius of 36.5 meters as pictured below:



The picture is drawn to scale with one centimeter in the picture representing 20 meters on an actual olympic track. The one of the eight lanes which is closest to the center of the track is called the first lane.

Below is an enlarged picture of one of the straight sections of the track with the blue line representing the line around the track with perimeter exactly 400 meters:



## Activity D

### Setting Up Sprinklers

From: <http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/C/8/tasks/607>

#### ***Next Generation Mathematics Learning Standard(s)***

**Define trigonometric ratios and solve problems involving right triangles.**

**GEO-G.SRT.8 Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems. ★**

**Note: Special right triangles refer to the 30-60-90 and 45-45-90 triangles.**

#### ***Mathematical Practice(s)***

MP4: Model with mathematics.

MP7: Look for and make use of structure.

#### ***Description***

This task involves several different types of geometric knowledge and problem solving: finding areas of sectors of circles, using trigonometric ratios to solve right triangles, and decomposing a complicated figure involving multiple circular arcs into parts whose areas can be found. It also engages students in some of the steps of the modeling cycle, in particular:

- formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between variables
- analyzing and performing operations on these relationships to draw conclusions
- interpreting the results of the mathematics in terms of the original situation.

#### ***Materials***

- Pencil/Pen
- Scrap Paper
- Compass
- Calculator

#### ***Focus questions for discussion***

- How can the Pythagorean theorem be used to solve problems involving right triangles?
- Is there a way to use the sector of a circle to find the sector's area?

### ***Extension***

Go to [desmos.com](https://desmos.com) and explore creating sectors. On this site you can explore the relationship between the area of circles, sector areas and sector angles. Use the “Challenges” to try your hand at estimation.

### ***Directions***

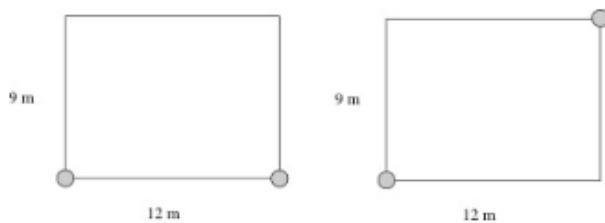
#### **Introduction**

1. Ask students about the processes one must go through when one has to decide between two courses of action. What factors must be considered? Is there a way to get assistance if that is what you want? Ask – “Think about a real-life situation that required you to decide between two choices. Describe what you went through before making your final decision.
2. Review the vocabulary terms related to this task (e.g. sector, arc, isosceles triangle).
3. Suggest that participants may want to begin by creating illustrations that outline “Selina’s” situation.
4. Thereafter, they should begin drawing conclusions and responding to questions “a”, “b”, and “c”, below.

## Task

Selina has a 9 meter by 12 meter lawn that she wants to water. She has two sprinklers, each of which can water grass within an 8.4-meter radius. Selina wants to set up the two sprinklers so that they are on corners of the lawn. She would like for the sprinklers to water as much of the lawn as possible, because she will have to manually water the part of the lawn that is not covered by the sprinklers.

a. Selina considers two different strategies for placing the sprinklers. One strategy is to put the sprinklers at opposite ends of a 12-meter side of the lawn. The other is to put the sprinklers at opposite corners of the lawn. (See the figures below.) Which strategy appears to be best? Justify your answer.



- b. If Selina chooses the better of these two strategies, what percentage of the lawn will she be able to water with the sprinklers?
- c. If Selina is allowed to put sprinklers in the interior of her lawn, how many sprinklers does she need to water the entire lawn?





## Activity E

# Unit Squares and Triangles

From: <http://tasks.illustrativemathematics.org/content-standards/HSG/CO/A/tasks/918>

### ***Next Generation Mathematical Learning Standard(s)***

**Use coordinates to prove simple geometric theorems algebraically.**

**GEO-G.GPE.4** On the coordinate plane, algebraically prove geometric theorems **and properties**.

Notes:

**Examples include but not limited to:**

- Given points and/or characteristics, prove or disprove a polygon is a specified quadrilateral or triangle based on its properties.
- Given a point that lies on a circle with a given center, prove or disprove that a specified point lies on the same circle.

**This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.**

**GEO-G.GPE.5** On the coordinate plane:

**GEO-G.GPE.5a Explore the proof** for the relationship between slopes of parallel and perpendicular lines;

**GEO-G.GPE.5b Determine if lines are parallel, perpendicular, or neither, based on their slopes;** and

**GEO-G.GPE.5c** Apply properties of parallel and perpendicular lines to solve geometric problems.

**Note: This standard is a fluency recommendation for Geometry. Fluency with the use of coordinates to establish geometric results and the use of geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields**

**Experiment with transformations in the plane.**

**GEO-G.CO.1** Know precise definitions of angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc **as these exist within a plane.**

**Prove theorems involving similarity.**

**GEO-G.SRT.5** Use congruence and similarity criteria for triangles to:

**GEO-G.SRT.5a** Solve problems **algebraically and geometrically**.

**GEO-G.SRT.5b** Prove relationships in geometric figures.

Notes: ASA, SAS, SSS, AAS, and Hypotenuse-Leg (HL) theorems are valid criteria for triangle congruence.  $AA\sim$ ,  $SAS\sim$ , and  $SSS\sim$  are valid criteria for triangle similarity.

**This standard is a fluency recommendation for Geometry. Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.**

***Mathematical Practice(s)***

MP4: Model with mathematics.

MP7: Look for and make use of structure.

***Description***

This problem provides an opportunity for a rich application of coordinate geometry. In the first solution, the coordinates of  $C$  are calculated explicitly by finding linear equations for the lines containing the sides of the triangles containing  $C$ . These coordinates then can be used to find the area of  $\triangle ABC$  using line segment  $AB$  as a base. Alternatively,  $\triangle ABC$  can be shown to be similar to the large right triangles in the picture. Since the large right triangles are each half of a 2 by 1 rectangle, their area is readily calculated. If the teacher wishes to emphasize the similarity approach, it may be necessary to provide some guidance. For example, labeling some of the other vertices ( $D, E, F$ ) and having students think about the angles in these triangles: more direct instructions can be provided if needed.

***Materials***

- Pencil/Pen
- Graph Paper
- Scrap paper
- Rulers
- Calculators

***Focus questions for discussion***

- How can we apply our knowledge of linear equations and (use graph paper) to assist us in determining the area of  $\triangle ABC$ ?
- We know the following – two lines are perpendicular with their slopes are inverse reciprocals of one another. How can we use this understanding in this task?

### Extension

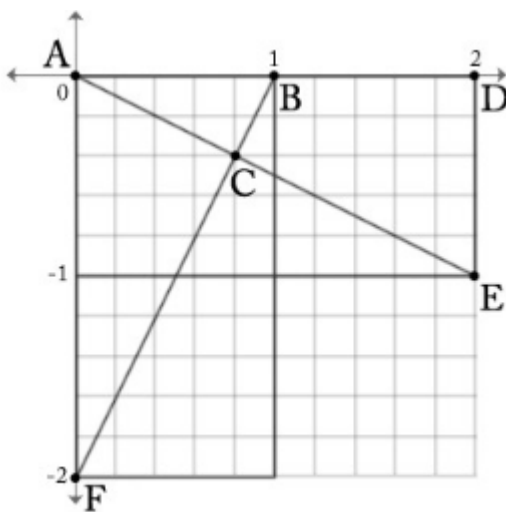
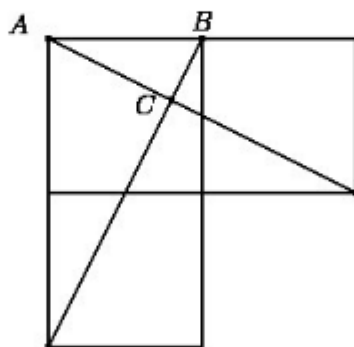
While working with partners, participants can recreate the task by changing the dimensions and exchanging their work.

### Directions

1. Provide students with supplies (i.e. graph paper, scrap paper, rulers, and calculators).
2. Go over the task with participants. Make sure they understand the dimensions of the figure. If necessary, clear up the term *three, unit squares* (each side of the smaller rectangles measures one unit.)
3. Have students proceed. Inform them that they should be mindful to incorporate the information that has already been calculated for them.

### Task

Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of  $\triangle ABC$ ?



## Activity F

# Unit Squares and Triangles

From: <http://tasks.illustrativemathematics.org/content-standards/HSG/SRT/A/3/tasks/1422>

### ***Next Generation Mathematics Learning Standard(s)***

**Understand similarity in terms of similarity transformations.**

**GEO-G.SRT.3** Use the properties of similarity transformations to establish the AA~, SSS~, and SAS~ criterion for two triangles to be similar.

### ***Mathematical Practice(s)***

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

### ***Description***

This task works toward establishing the AA (Angle-Angle) criterion for similarity of triangles by providing a detailed sequence of transformations that moves one of the given triangles to the other. In general, a reflection may be necessary to illustrate AA for a pair of triangles, though this is unnecessary for the given triangles (they have the same "orientation").

### ***Materials***

- Pencil/Pen
- Graph Paper
- Scrap paper

### ***Focus questions for discussion***

- What are some of the potential possibilities for sequences of transformations?
- Does the order of the transformations make a difference?

### ***Extension***

Students can make up their own transformations experimenting by changing the sequence of the transformations. Students may also want to try dilations and vertical scaling if they did not use those methods during this task.

### ***Directions***

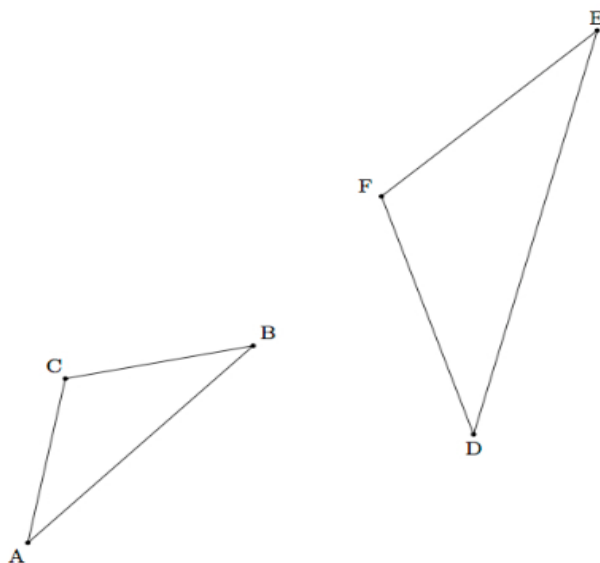
#### ***Introduction***

The AA criterion for triangle similarity is important in the same way the ASA, SAS, and SSS criteria for triangle congruence are important. Producing an explicit series of transformations exhibiting a similarity can be difficult so, when we have knowledge about the angles in triangles this is a very convenient tool.

1. Let the students begin by observing the triangles' proximity to each other.
2. Students who are working in pairs can share with the partners some of the possible ways to "overlap" one triangle over the other and determine whether or not they are symmetrical.
3. Provide them with paper and have them begin the process of translating the figures.

### Task

In the two triangles pictured below  $m(\angle A) = m(\angle D)$  and  $m(\angle B) = m(\angle E)$ :



Using a sequence of translations, rotations, reflections, and/or dilations, show that  $\triangle ABC$  is similar to  $\triangle DEF$ .

## Activity G

# Origami Silver Rectangle

From: <http://tasks.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/1488>

### ***Next Generation Mathematics Learning Standard(s)***

**Understand congruence and similarity using physical models, transparencies or geometry software.**

**NY-8.G.1** Verify experimentally the properties of rotations, reflections, and translations.

### **Notes:**

A translation displaces every point in the plane by the same distance (in the same direction) and can be described using a vector.

A rotation requires knowing the center/point of rotation and the measure/direction of the angle of rotation.

A line reflection requires a line and the knowledge of perpendicular bisectors

**Make geometric constructions.**

**GEO-G.CO.12** Make, **justify and apply** formal geometric constructions.

### **Notes:**

**Examples** of constructions **include but are not limited to:**

- Copy segments and angles.
- Bisect segments and angles.
- Construct perpendicular lines including through a point on or off a given line.
- Construct a line parallel to a given line through a point not on the line.
- Construct a triangle with given lengths.
- **Construct points of concurrency of a triangle (centroid, circumcenter, incenter, and orthocenter).**
- **Construct the inscribed circle of a triangle.**
- **Construct the circumscribed circle of a triangle.**
- **Constructions of transformations. (see GEO-G.CO.5)**

**This standard is a fluency recommendation for Geometry. Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.**

### ***Mathematical Practice(s)***

MP2: Reason abstractly and quantitatively.

MP5: Use appropriate tools strategically.

### ***Description***

The purpose of this task is to apply geometry in order analyze the shape of a rectangle obtained by folding paper. The central geometric ideas involved are reflections (used to model the paper folds), analysis of angles in triangles, and the Pythagorean Theorem. The task is appropriate either at the 8th

grade level or in high school: the only difference would be the level of rigor expected in the explanation.

#### Materials

- Pencil/Pen
- Paper (for folding)
- Scrap paper
- Rulers (straight edges)
- Directions for folding paper
- Optional – access to [http://en.wikipedia.org/wiki/Silver\\_ratio](http://en.wikipedia.org/wiki/Silver_ratio)

#### ***Focus questions for discussion***

- The triangles that will be created are all of the 45-45-90 variety. As a result, how can we use the scale factor between any of these triangles to determine the dimensions of the rectangle?
- We know that reflection over a diagonal is a symmetry of a square. How can this knowledge assist us with this task?

#### ***Directions***

1. If participants do not have Internet access, provide them with background information on *silver ratios*. (“This was one of three rectangles identified in ancient times as having important properties (the bronze rectangle has a side ratio of  $(\sqrt{3}, 1)$  and the golden rectangle has a side ratio of  $(1 + \sqrt{5}:2)$ . Each of these three rectangles can be constructed by folding paper.
2. At this time, the facilitator should go through the different steps of the construction and then prompt students to examine the ratio of side lengths in the final rectangle.
3. After going through the different steps with the participants, direct the students to questions “a”, “b”, and “c.”

## Task

This task examines the mathematics behind an origami construction of a rectangle whose sides have the ratio ( $\sqrt{2} : 1$ ). Such a rectangle is called a *silver rectangle*.

Beginning with a square piece of paper, first fold and unfold it leaving the diagonal crease as shown here:



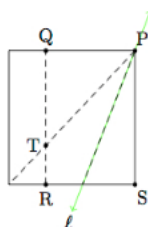
Next fold the bottom right corner up to the diagonal:



After unfolding then fold the left hand side of the rectangle over to the crease from the previous fold:



Here is a picture, after the last step has been unfolded, with all folds shown and some important points marked. In the picture  $T$  is the reflection of  $S$  about  $\ell$ .



- Suppose  $s$  is the side length of our square. Show that  $|PT| = s$ .
- Show that  $\triangle PQT$  is a 45-45-90 isosceles triangle.
- Calculate  $|PQ|$  and conclude that  $PQRS$  is a silver rectangle.